تعداد سؤالات: تستی: تشریحی: ۵
زمان آزمون (دقیقه): تستی: -- تشریحی: ۱۲۰
نام درس: مقاومت مصالح۲
رشته تحصیلی /کد درس : مهندسی عمران

بارم هر سوال ۲/۸۰ می باشد.

۱-

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\left.\begin{matrix}\sigma_1\\ \sigma_2\end{matrix}\right\} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{40 - (-100)}{2}\right)^2 + (-50)^2}$$

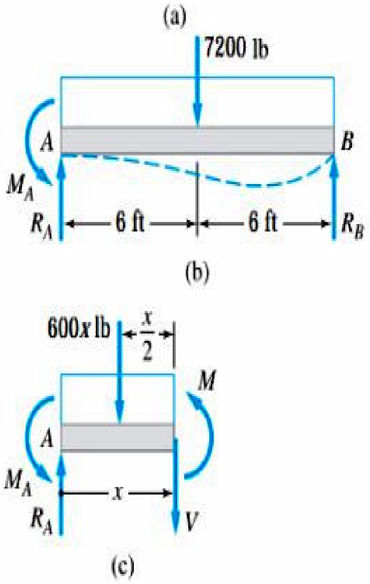
$$= 86.0 \text{ MPa}$$

$$\tan 2\theta = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{40 - (-100)}{2(-50)} = 1.400$$

$$2\theta = 54.46^\circ \quad \text{and} \quad 54.46^\circ + 180^\circ = 234.46^\circ$$

$$\theta = 27.23^\circ \quad \text{and} \quad 117.23^\circ$$

-۲



$$\Sigma F_y = 0 \quad +\uparrow \quad R_A + R_B - 7200 = 0 \qquad \text{(a)}$$

$$\Sigma M_A = 0 \quad +\circlearrowleft \quad M_A + R_B(12) - 7200(6) = 0 \qquad \text{(b)}$$

Because there are three support reactions (R_A, R_B, and M_A) but only two independent equilibrium equations, the degree of static indeterminacy is one.

Compatibility A third equation containing the support reactions is obtained by analyzing the deformation of the beam. We start with the expression for the bending moment, obtainable from the free-body diagram in Fig. (c):

$$M = -M_A + R_A x - 600x\left(\frac{x}{2}\right) \text{ lb}\cdot\text{ft}$$

Substituting M into the differential equation for the elastic curve and integrating twice, we get

$$EIv'' = -M_A + R_A x - 300x^2 \text{ lb}\cdot\text{ft}$$

$$EIv' = -M_A x + R_A\frac{x^2}{2} - 100x^3 + C_1 \text{ lb}\cdot\text{ft}^2$$

$$EIv = -M_A\frac{x^2}{2} + R_A\frac{x^3}{6} - 25x^4 + C_1 x + C_2 \text{ lb}\cdot\text{ft}^3$$

Since there are three support reactions, we also have three support constraints. Applying these constraints to the elastic curve, shown by the dashed line in Fig. (b), we get

1. $v'|_{x=0} = 0$ (no rotation at A) $\qquad C_1 = 0$
2. $v|_{x=0} = 0$ (no deflection at A) $\qquad C_2 = 0$
3. $v|_{x=L} = 0$ (no deflection at B)

$$-M_A\frac{(12)^2}{2} + R_A\frac{(12)^3}{6} - 25(12)^4 = 0 \qquad \text{(c)}$$

The solution of Eqs. (a)–(c) is

$$M_A = 10\,800 \text{ lb}\cdot\text{ft} \qquad R_A = 4500 \text{ lb} \qquad R_B = 2700 \text{ lb}$$ *Answer*

تعداد سؤالات: تستی: تشریحی: ۵ | زمان آزمون (دقیقه): تستی: -- تشریحی: ۱۲۰

نام درس: مقاومت مصالح ۲

رشته تحصیلی /کد درس : مهندسی عمران

-۳

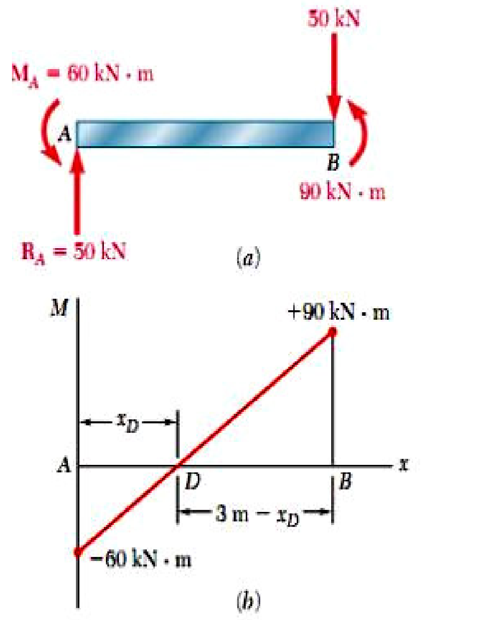


Fig. 9.45

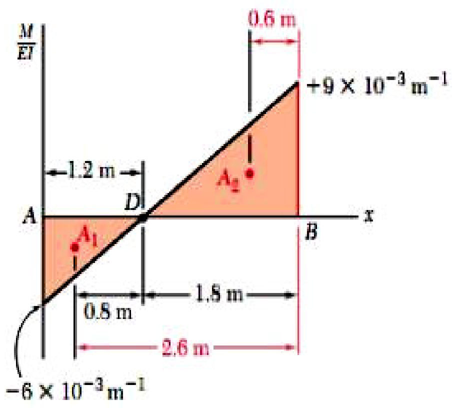


Fig. 9.46

$$\frac{x_D}{60} = \frac{3 - x_D}{90} = \frac{3}{150} \qquad x_D = 1.2\text{ m}$$

Dividing by the flexural rigidity EI the values obtained for M, we draw the (M/EI) diagram (Fig. 9.46) and compute the areas corresponding respectively to the segments AD and DB, assigning a positive sign to the area located above the x axis, and a negative sign to the area located below that axis. Using the first moment-area theorem, we write

$$\begin{aligned}\theta_{B/A} = \theta_B - \theta_A &= \text{area from } A \text{ to } B = A_1 + A_2\\ &= -\tfrac{1}{2}(1.2\text{ m})(6 \times 10^{-3}\text{ m}^{-1}) + \tfrac{1}{2}(1.8\text{ m})(9 \times 10^{-3}\text{ m}^{-1})\\ &= -3.6 \times 10^{-3} + 8.1 \times 10^{-3}\\ &= +4.5 \times 10^{-3}\text{ rad}\end{aligned}$$

and, since $\theta_A = 0$,

$$\theta_B = +4.5 \times 10^{-3}\text{ rad}$$

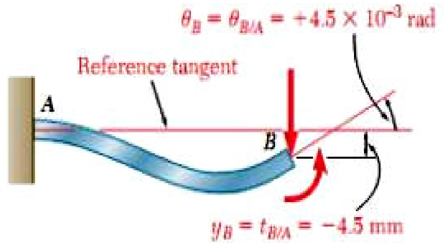
Using now the second moment-area theorem, we write that the tangential deviation $t_{B/A}$ is equal to the first moment about a vertical axis through B of the total area between A and B. Expressing the moment of each partial area as the product of that area and of the distance from its centroid to the axis through B, we have

$$\begin{aligned}t_{B/A} &= A_1(2.6\text{ m}) + A_2(0.6\text{ m})\\ &= (-3.6 \times 10^{-3})(2.6\text{ m}) + (8.1 \times 10^{-3})(0.6\text{ m})\\ &= -9.36\text{ mm} + 4.86\text{ mm} = -4.50\text{ mm}\end{aligned}$$

Since the reference tangent at A is horizontal, the deflection at B is equal to $t_{B/A}$ and we have

$$y_B = t_{B/A} = -4.50\text{ mm}$$

The deflected beam has been sketched in Fig. 9.47.



-۴

$$I = 15.588 \times 10^{-6}\ \text{m}^4$$

$$\frac{a^4}{12} = 15.588 \times 10^{-6} \qquad a = 116.95\ \text{mm}$$

The value of the normal stress is

$$\sigma = \frac{P}{A} = \frac{200\ \text{kN}}{(0.11695\ \text{m})^2} = 14.62\ \text{MPa}$$

Since this value is larger than the allowable stress, the dimension obtained is not acceptable, and we must select the cross section on the basis of its resistance to compression. We write

$$A = \frac{P}{\sigma_{\text{all}}} = \frac{200\ \text{kN}}{12\ \text{MPa}} = 16.67 \times 10^{-3}\ \text{m}^2$$

$$a^2 = 16.67 \times 10^{-3}\ \text{m}^2 \qquad a = 129.1\ \text{mm}$$

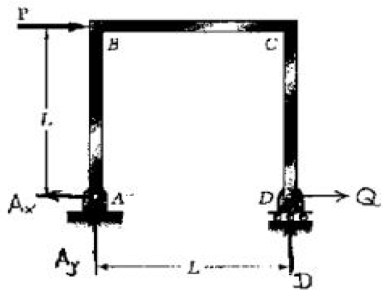
A 130 × 130-mm cross section is acceptable.

تعداد سؤالات: تستی: — **تشریحی:** ۵   **زمان آزمون (دقیقه): تستی:** -- **تشریحی:** ۱۲۰

نام درس: مقاومت مصالح ۲

رشته تحصیلی /کد درس : مهندسی عمران

۵-



Add dummy force Q at point D as shown.

Statics $\circlearrowleft \Sigma M_A = 0: \; DL - PL = 0 \qquad D = P \uparrow$

$\overset{+}{\rightarrow} \Sigma F_x = 0: \; -A_x + P + Q = 0 \qquad A_x = (P+Q) \leftarrow$

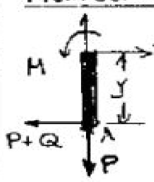
$+\uparrow \Sigma F_y = 0: \; A_y + D = 0 \qquad A_y = P \downarrow$

$U = U_{AB} + U_{BC} + U_{CD}$

By Castigliano's theorem, $\delta_D = \dfrac{\partial U}{\partial Q}$

$$\delta_D = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q} + \frac{\partial U_{CD}}{\partial Q}$$

Member AB

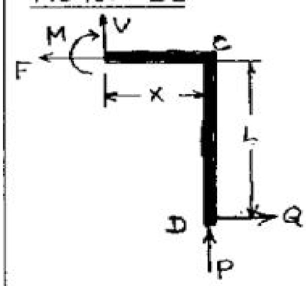


$M = (P+Q)y \qquad \dfrac{\partial M}{\partial Q} = y \qquad$ Set $Q = 0 \qquad M = Py$

$$U_{AB} = \int_0^L \frac{M^2\,dy}{2EI}$$

$$\frac{\partial U_{AB}}{\partial Q} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial Q}\,dy = \frac{P}{EI}\int_0^L y^2\,dy = \frac{PL^3}{3EI}$$

Member BC



$M = Px + QL \qquad \dfrac{\partial M}{\partial Q} = L \qquad$ Set $Q=0 \qquad M = Px$

$$U_{BC} = \int_0^L \frac{M^2\,dx}{2EI}$$

$$\frac{\partial U_{BC}}{\partial Q} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial Q}\,dx = \frac{PL}{EI}\int_0^L x\,dx = \frac{PL^3}{2EI}$$

Member CD $\quad M = Qy \qquad \dfrac{\partial M}{\partial Q} = y \qquad$ Set $Q = 0 \qquad M = 0$

$$U_{CD} = \int_0^L \frac{M^2\,dy}{2EI} \qquad \frac{\partial U_{CD}}{\partial Q} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial Q}\,dy = 0$$

$$\delta_D = \frac{PL^3}{3EI} + \frac{PL^3}{2EI} + 0 = \frac{5PL^3}{6EI} \rightarrow$$